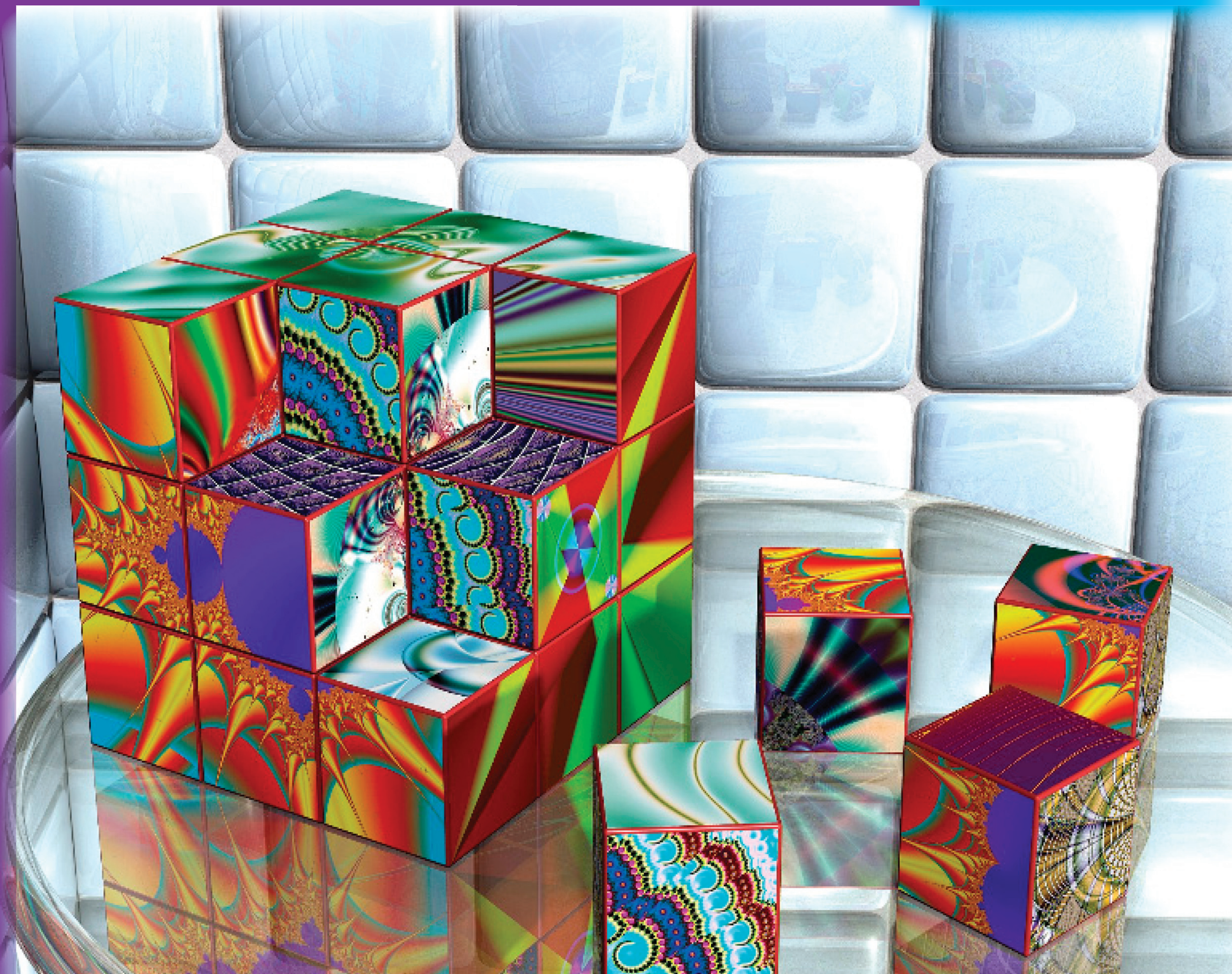
educàlia editorial

Rosario Carrasco Torres

MATHEMATICS 3 ESO



MATHEMATICS 3 ESO

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REMEMBER THAT:

	CARDINAL	ORDINAL		CARDINAL	ORDINAL
1	One	First (1st)	21	Twenty-one	Twenty-first (21th)
2	Two	Second (2nd)	22	Twenty-two	Twenty-second (22th)
3	Three	Third (3rd)	23	Twenty-three	Twenty-third (23th)
4	Four	Fourth (4th)	24	Twenty-four	Twenty-fourth (24th)
5	Five	Fifth (5th)	25	Twenty-five	Twenty-fifth (25th)
6	Six	Sixth (6th)	26	Twenty-six	Twenty-sixth (26th)
7	Seven	Seventh (7th)	27	Twenty-seven	Twenty-seventh (27th)
8	Eight	Eighth (8th)	30	Thirty	Thirtieth (30th)
9	Nine	Ninth (9th)	40	Forty	Fortieth (40th)
10	Ten	Tenth (10th)	50	Fifty	Fiftieth (50th)
11	Eleven	Eleventh (11th)	100	One hundred	Hundredth
12	Twelve	Twelfth (12th)	1000	One thousand	Thousandth
13	Thirteen	Thirteenth (13th)	100000	One hundred thousand	Hundred thousandth
20	Twenty	Twentieth (20th)	1000000	One million	Millionth

Expressing numbers in English:

* If a number is between 21 and 99, and the second digit is not zero, we should write the number as two words separated by a hyphen.

Examples : 34 ≡ thirty-four; 62 ≡ sixty-two; 98 ≡ ninety-eight

* If a number is over 100 it is usually written in figures. However, if you want to write it in words you must put the word "**and**" before the last two figures.

Examples: 325 ≡ three hundred **and** twenty-five; 709 ≡ seven hundred **and** nine

* If a number is **between** 1000 and 1.000.000 is usually written **using commas.**

Examples: 1.306 ≡ one thousand, three hundred and six
4.512 ≡ four thousand, five hundred and twelve
6.305.215 ≡ six million, three hundred and five thousand, two hundred and fifteen

Remember that in English they use commas instead of **points** to express numbers greater than 999

Note :

They don't put "and" in American English!

Generally speaking, the word "**and**" is used after every hundred digit of a number. Examples:
512 ≡ five hundred **and** twelve
2.340.000 ≡ two million, three hundred **and** forty thousand

* If a number is a four-figure number ending in 00 it could be said or sometimes written as a number of hundreds.

Examples: 1.600 ≡ "sixteen hundred"
2.700 ≡ "twenty-seven hundred"

Special cases

2 ≡ couple or pair
6 ≡ half a dozen
12 ≡ a dozen (used mostly in commerce)
1.000.000.000.000 ≡ billion

Note :

Long time ago, British speakers used " billion " to mean a million million like in Spanish. It means 1.000.000.000.000

However, nowadays they usually use it to mean a thousand million (or milliard), like American speakers. It means 1.000.000.000

Saying telephone numbers

Each number is usually said separately. If a telephone number contains a double number, we use the word "double". Sometimes we group them and introduce pauses in between.

Example: 644 012 839 ≡ six four four "oh" one two eight three nine

Saying time of day

Let's see some examples:

7:00 a.m. *or* seven o'clock in the morning
3:30 p.m. *or* half past three in the afternoon
8:15 p.m. *or* a quarter past eight in the evening

Saying dates

The day usually comes before the month and the ordinal suffix is always vocalised and most of the times it is appended.

Example: 2^{nd} May 1967

Curiously, when we write it down or when we say it we normally put "the" and "of" like:

Example: The second of May …

Years

A year is usually said in two parts.

Examples: 1997 ≡ nineteen ninety-seven
2015 ≡ twenty fifteen

★Except 2000 which is normally said *two thousand*

If the year ends in 00 we usually say *hundred*

* If the year ends in *01* to *09* we usually say it like a number.

Example: 2004 ≡ two thousand and four

Percentages

You can say "per cent " or "percent" after the number.

Example: 67% ≡ sixty seven per cent

Fractions

There are several ways to say a fraction in English.

Example: 1/8 ≡ one over eight

one – eighth

one *divided by* eight

You use ordinals for the denominator and place a hyphen in between.

* If it is necessary we use **plural** in the second part.

Examples: 3/6 ≡ three sixth**s**
2/3 ≡ two third**s**

* There are some special cases:

½ ≡ one half *or* a half

What is more, we can say it every time it appears. For instance:

4½ ≡ four and a half

¼ ≡ a quarter

Decimal numbers

➢ In decimal numbers, remember that:

- In Spain, we use a comma and in Britain, they use a point.
- After a comma, every digit is said separately.
- The figure 0 is usually called *nought* before a comma and *oh* after the comma. Example: 0,203≡ nought comma two oh three

Remember that in English they use points instead of commas to express decimal numbers

UNIT 1: " RATIONAL NUMBERS "

KEY VOCABULARY:

Fraction
Numerator
Denominator
Whole
Proper fraction
Improper fraction
Number line
Equivalent fractions
To amplify
To simplify
Simplest form
To equalize
Unlike denominators
Decimal number
Exact decimal number
Terminating decimal number
Period
Recurring decimal number
Repeating decimal number
Irrational number
Rational number

In this unit you will learn how to:

- *Identify a fraction*
- *Calculate equivalent fractions*
- *Simplify a fraction to its simplest form*
- *Operate with fractions*
- *Calculate the fraction form of a given decimal number*
- *Identify a Rational Number*

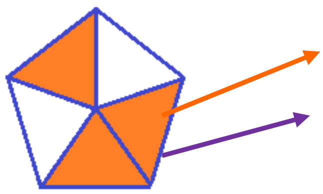
1. Fractions: definition, interpretations and types

A fraction is an expression that indicates the quotient of two integer numbers.

It is normally expressed in the form $\frac{a}{b}$ where the top part (**a**) is called **numerator** and the bottom part (**b**) is called **denominator** who is a *non-zero integer*.

Remember that: the denominator indicates how many parts the whole is divided into and the numerator indicates how many parts you take or have.

Example :



$\frac{3}{5}$

You can also see a fraction as an **operator**:

Example:

$$\frac{3}{5} \text{ of } 45 = \frac{3}{5} \cdot 45 = \frac{3 \cdot 45}{5} = \frac{135}{5} = 27$$

Let's see two types of fractions:

1.1. Proper fractions

In a proper fraction: Numerator < Denominator. Example: $\frac{4}{7}$

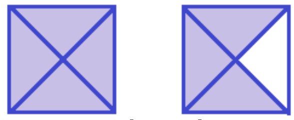
Remember that: $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$

1.2. Improper fractions

In an improper fraction: Numerator $\geq$ Denominator. Example: $\frac{8}{3}$

- As improper fractions are more than a whole, we need more than a unit to represent them.

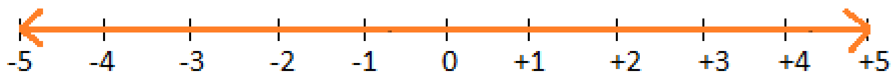
  Example: " Represent the fraction 7/4 ". →



  As the whole is divided in four parts we need to represent it using two drawings.

2. Representing fractions on the Number line

We already know how to represent Integers on the Number line:



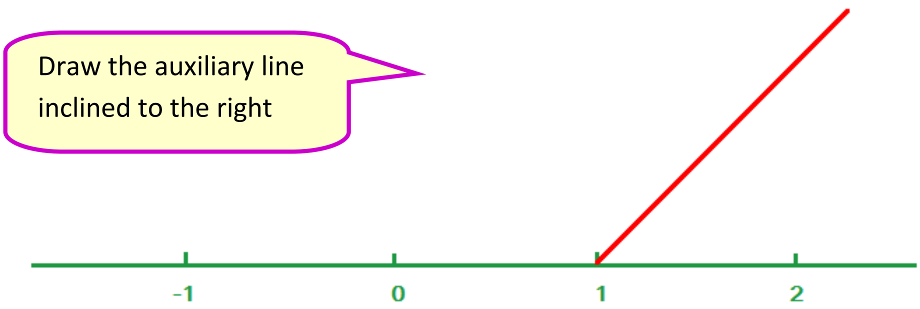
We can represent fractions on the number line in the same way as we learnt to represent Integer numbers. Let's see how to do it through some examples:

Example 1: " Represent $\frac{7}{5}$ on the Number line. "

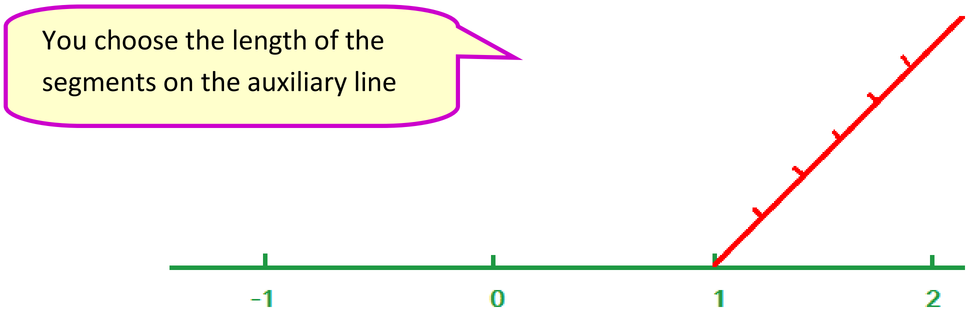
1st. step: Divide numerator by denominator

7	5
2	1

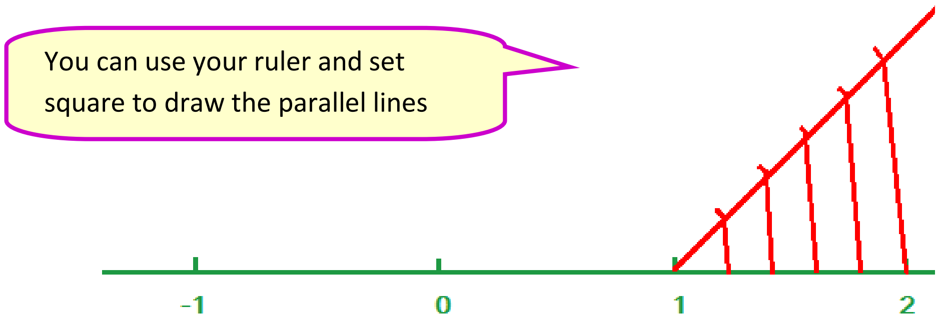
2nd. step: Draw a line with origin on the number obtained as quotient of the previous division.



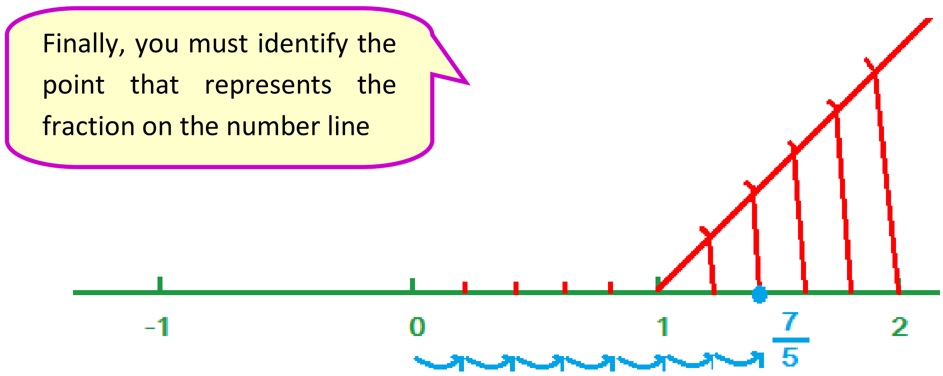
3rd. step: Make on that line as many equal segments as the denominator of the fraction indicates.



4th. step: Draw a line between the endpoint of the last segment and the next integer number (in this case the number 2) and draw parallel lines at the end of all of the segments you've drawn.



5th. step: At this point, we can see all the unit segments divided into the same number of parts and the only thing we have to do is to count since zero the number of parts indicated by the numerator:

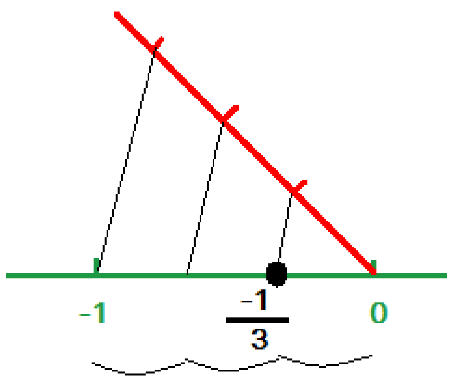


Example 2: " Represent $-\frac{1}{3}$ on the Number line. "

Dividing numerator by denominator

1 | 3
0,

As we can see, in this case the fraction is negative and quotient starts with 0 therefore the auxiliary line must be drawn since zero to the left:

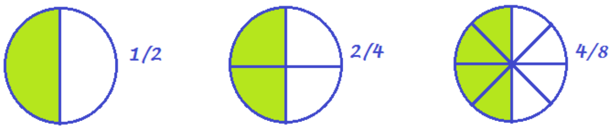


In conclusion, at this point we can assure that we know how to represent every type of fraction on the Number line.

3. Equivalent fractions

Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent when $a \cdot d = b \cdot c$. When two fractions are equivalent, although they appear differently, they have the same value.

Example 1:



Notice that the shaded area s the same

In this particular case, their value is: $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = 0{,}5$

Example 2: " Are the fractions $\frac{3}{4}$ and $\frac{21}{28}$ equivalent ? "

To check it we only have to multiply in a cross ⟶ $3 \cdot 28 \stackrel{?}{=} 4 \cdot 21$

Therefore, the fractions are equivalent. $84 \quad 84$

- You can also be asked about one of the terms of a fraction in order to make it equivalent to another one.

Example: " Given the fraction $\frac{7}{8}$ and the number 14, construct with it another fraction to make it equivalent to the first one".

The problem is to find out a number to complete the proportion: $\frac{7}{8} = \frac{14}{\boxed{?}}$

We can see it like an equation!

If we call that number **x**, the thing is to calculate $8 \cdot 14 = 7 \cdot \mathbf{x} \longrightarrow \mathbf{x} = \frac{112}{7} = 16$

- There are two ways to calculate equivalent fractions to one given. Let' see them:

3.1. Amplifying fractions

To amplify a fraction you only have to multiply its numerator and its denominator by the same non-zero integer number.

$$\frac{a}{b} = \frac{a \cdot k}{b \cdot k}, \qquad k \neq 0$$

Example: " Amplify the fraction $\frac{3}{2}$ to obtain four equivalent fractions to the given"

For instance, we can multiply it by 4 $\longrightarrow \frac{3\cdot4}{2\cdot4}=\frac{12}{8} \longrightarrow \frac{3}{2}=\frac{12}{8}$, and you can even construct a chain of equivalent fractions multiplying each one of them by any non-zero integer number as it is shown below.

$$\frac{3}{2} \overset{\cdot 4}{=} \frac{12}{8} \overset{\cdot 2}{=} \frac{24}{16} \overset{\cdot 3}{=} \frac{72}{48} \overset{\cdot 5}{=} \frac{360}{240}$$

The chain **CAN** continue forever

3.2. Simplifying fractions

To simplify a fraction you can choose one of the following methods:

1st. method: Dividing its numerator and its denominator by the same non-zero integer number.

$$\frac{a}{b}=\frac{a:k}{b:k}, \quad k \neq 0$$

Example: " Simplify the fraction $\frac{250}{450}$ to obtain three equivalent fractions to the given"

$$\frac{250}{450} \overset{:2}{=} \frac{125}{225} \overset{:5}{=} \frac{25}{45} \overset{:5}{=} \frac{5}{9}$$

The chain **CAN'T** continue forever.

2nd. method: Dividing the numerator and the denominator in the fraction by the GCD of both of them.

$$\frac{a}{b}=\frac{a : GCD(a,b)}{b : GCD(a,b)}$$

Example: " Simplify the fraction $\frac{250}{450}$ "

$$\frac{250}{450} = \frac{250 : GCD(250,450)}{450 : GCD(250,450)} = \frac{250:50}{450:50} = \frac{5}{9}$$

3.3. Simplest form of a fraction

The **simplest form** of a fraction is a fraction whose numerator and denominator have no common factor except 1. Example: "Obtain the simplest form of $\frac{25}{15}$ "

$\frac{25}{15}=\frac{25:5}{15:5}=\frac{5}{3}$ ⟸ This is the simplest form because 5 and 3 have no common factors.

- The key to obtain *the simplest form using the fastest way* is the second method to simplify fractions explained above. It means calculating the GCD of numerator and the denominator of the fraction. Using that method you always obtain the simplest fraction in only one step.

Examples: "Obtain the simplest form of the fractions: $\frac{45}{36}$ and $\frac{100}{75}$ "

ONLY ONE STEP!

$$\frac{45}{36}=\frac{45:9}{36:9}=\frac{5}{4} \qquad \frac{100}{75}=\frac{100:25}{75:25}=\frac{4}{3}$$

GCD(45, 36) = 9 GCD(100, 75) = 25

4. Equalizing denominators

To equalize denominators is to make unlike denominators equal. The process consists in obtaining equivalent fractions to the given whose denominators are the same.

To reduce a set of fractions to the same denominator, you must follow the next steps:

1st. step: Calculate the LCM of the correspondent denominators
2nd. step: Divide the LCM by each one of the original denominators
3rd. step: Multiply the number you have obtained in the previous step by each one of the original numerators.

Example: " Given the fractions $\frac{3}{25}, \frac{-2}{15}, \frac{7}{3}$ equalize their denominators".

1st. step: LCM(25, 15, 3) = 75

2nd. step: 75 : 25 = 3 75: 15 = 5 75 : 3 = 25

3rd. step: $\frac{3}{25}=\frac{3\cdot 3}{25\cdot 3}=\frac{9}{75}$ $\frac{-2}{15}=\frac{-2\cdot 5}{15\cdot 5}=\frac{-10}{75}$ $\frac{7}{3}=\frac{7\cdot 25}{3\cdot 25}=\frac{175}{75}$

With that process we have obtained three equivalent fractions with the same denominator which are: $\frac{9}{75}$, $\frac{-10}{75}$ and $\frac{175}{75}$

5. Comparing fractions

To compare fractions means to order them in an ascending or descending order.

- It is obvious that if the fractions have **the same denominator**, the problem is very simple because we only have to look at the numerators.

Example: " Order these fractions in an ascending order: $\frac{3}{5}$, $\frac{-1}{5}$ and $\frac{6}{5}$ ".

Obviously: $\frac{-1}{5}<\frac{3}{5}<\frac{6}{5}$

- When the fractions have **unlike denominators**, we must equalize them first.

Example: " Order these fractions in an ascending order: $\frac{3}{25}, \frac{-2}{15}, \frac{7}{3}$ ".

We must equalize first but, fortunately in this case, if we take a closer look, we notice that they are the same fractions from the example about equalizing fractions, then we already know that :

$$\frac{3}{25} = \frac{9}{75} \qquad \frac{-2}{15} = \frac{-10}{75} \qquad \frac{7}{3} = \frac{175}{75}$$

It is easier to look at the equivalent fractions!

Therefore, we can order our original fractions like:

$$\frac{-2}{15} < \frac{3}{25} < \frac{7}{3}$$

The negative one is the lowest one!

6. Operations with fractions

There are several operations that can be performed with fractions. Let's see them:

6.1. Addition and subtraction

There are **two cases**:

a) If the fractions have the same denominators you only have to operate with the numerators.

Example: " Calculate: $\frac{-2}{5} + \frac{3}{5} - \frac{7}{5} =$ ".

As they have the same denominator → $\frac{-2}{5} + \frac{3}{5} - \frac{7}{5} = \frac{8}{5}$

b) If the fractions have different denominators, first of all we have to reduce them to the same denominator.

Example 1: " Calculate: $\frac{-2}{3} + \frac{3}{5} - \frac{7}{45} =$ ".

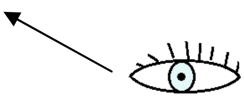
As they have the unlike denominators → $\frac{-2}{3} + \frac{3}{5} - \frac{7}{45} = \frac{-30}{45} + \frac{27}{45} - \frac{7}{45} = \frac{-10}{45} = \frac{-2}{9}$

Reducing to the same denominator

Remember to simplify

Example 2: " Calculate: $\frac{5}{8} + 3 - \frac{11}{12} =$ ".

$$\frac{5}{8} + 3 - \frac{11}{12} = \frac{5}{8} + \frac{3}{1} - \frac{11}{12} = \frac{15}{24} + \frac{72}{24} - \frac{22}{24} = \frac{15+72-22}{24} = \frac{65}{24}$$



6.2. Multiplication and division

- **To multiply** two fractions you only have to multiply denominators and denominators respectively (horizontally).

Example 1: $\frac{3}{4} \cdot \frac{2}{15} = \frac{6}{60} = \frac{1}{110}$

Example 2: $\frac{5}{3} \cdot 6 = \frac{5}{3} \cdot \frac{6}{1} = \frac{30}{3} = 10$

- **To divide** two fractions you only have to multiply their terms in a cross.

Example: $\frac{4}{5} : \frac{2}{7} = \frac{28}{10} = \frac{14}{5}$

Example: $6 : \frac{2}{5} = \frac{6}{1} : \frac{2}{5} = \frac{30}{2} = 15$

6.3. BIDMAS

The key to solve combined operations with fractions is BIDMAS with is an acronym that reminds us the correct order to do those operations.

Braquets
Indices
Divisions } ← Same level
Multiplications
Additions } ← Same level
Subtractions

Remember that: For the operations in the same level, do first what you see first (from the right to the left).

Example 1: " Solve the following operations: "

$$\frac{1205}{8} \cdot \frac{2}{10} : \frac{4}{3} + \frac{3}{2} - \frac{5}{12} = \frac{10}{80} : \frac{4}{3} + \frac{3}{2} - \frac{5}{12} = \frac{30}{120} + \frac{3}{2} - \frac{5}{12} = \frac{30+180-50}{120} = = \frac{160}{120} = \frac{4}{3}$$

Same level — LCM (120, 2, 12) = 120 — Always simplify

Example 2: " Solve the following operations: "

$$\frac{5}{2} - 6 - \left(\frac{3}{4} : \frac{15}{8}\right) = \frac{5}{2} - 6 - \left(\frac{24}{60}\right) = \frac{150 - 360 - 24}{60} = \frac{-234}{60} = \frac{-117}{30} = \frac{-39}{10}$$

Braquets — LCM (2, 60) = 60 — Always simplify

7. Decimal numbers

A **decimal number** is a linear array of numbers expressed in two parts separated by a comma. The first part (left part) is called the **whole part** and the second part (right part) is called the **decimal part**.

Remember that in English it should be a point

whole part **,** *decimal part*

Remember that: every place you move to the right you get ten times smaller and every time you move to the left you get 10 times greater.

Example : " Underline the whole part and in the following decimal numbers :

56,023 ; 102,5 ; 0,00034 ; 4678,9 ; 1,111

➢ ONE TIP: To identify each one of the digits of a decimal number you can think of the following diagram:

THOUSANDTHS	HUNDREDTHS	UNITS **,**	TENTHS	HUNDREDTHS	THOUSANDTHS	TEN THOUSANTHS

Example: " How do you write the following number ?:

* 23, 867 → Twenty-three ones and eight hundred and sixty-seven thousandths
→ Twenty-three comma eight six seven

Both expressions are correct

7.1. Types of decimal numbers

There are four types of decimal numbers:

I. **Exact** or **Terminating** decimal number: its decimal part is formed by a finite set of numbers. It means that you can write down all its digits because it doesn't go forever. Examples: 3,25 ; 10,444 ; 6,789 ; 0,00045 ; 1,99999

II. **Recurring** or **Repeating** decimal number: its decimal part goes forever and there is a digit or a group of digits called **period** that is repeated endlessly.

➢ There are two kinds of recurring decimal numbers:

Pure Recurring decimal number: the period is just after the comma.

Mixed Recurring decimal number: the decimal part is formed by two parts: one part before the period and the period itself.

Example: 7,88888... ; 0,341212... ; 0,020202...

Pay attention to the dots

★We usually write them in a shorter form using a **curved bar** on top of the period.

Example: 7,88888...= $7,\overset{\frown}{8}$; 0,341212... = $0,34\overset{\frown}{12}$; 0,0202... = $0,\overset{\frown}{02}$

III. **Irrational** number: its decimal part goes forever but there is no group of digits repeated. Examples: 0,12345… ; 4,01001001… ; 010203…

7.2. From a fraction to a decimal number

To express a fraction as a decimal number we only have to divide its numerator by its denominator.

- Nevertheless, if we simplify the fraction and take a look at the denominator we can predict the kind of decimal number . In this case there are three possibilities:

1st case: if the numerator is multiple of the denominator we obtain an **integer** number.
2nd case: if the denominator only has as factors 2 or 5 or both numbers then it is an **exact** decimal number.
3rd case: any other possibility yields a **recurring** decimal number.

Example: " Try to guess what kind of decimal number yield the following fractions:

$\frac{93}{3}$ ⟵ It is integer number because 93 is multiple of 3

$\frac{21}{35}=\frac{3}{5}$ ⟵ After simplifying, it is 5 the only factor then it is an exact number

$\frac{3}{210}=\frac{1}{70}=\frac{1}{2\cdot 5\cdot 7}$ ⟵ Apart from 2 and 5 there is also another factor, 7.

Then, it is a recurring number.

7.3. From a decimal number to its "corresponding fraction"

The process depends on the kind of decimal number as the following table shows:

TYPE	NUMERATOR	DENOMINATOR	Examples
Exact	The number without the comma.	The number one followed by so many zeros as decimal digits the number has.	$4,35=\frac{435}{100}$
Pure Recurring	The number without the comma and the bar of the period minus the whole part.	So many nines as digits the period has.	$8,\overset{\frown}{3}=\frac{83-8}{9}=\frac{75}{9}$
Mixed recurring	The number without the comma and the bar of the period minus the whole part followed by the part before the period.	So many nines as digits the period has followed by so many zeros as decimal digits before the period it has.	$1,7\overset{\frown}{42}=\frac{1742-17}{990}=\frac{1725}{990}$

8. The set of Rational Numbers

All the numbers that can be expressed as a fraction form the set of Rational numbers and we use $\mathbb{Q}$ to represent it. It obviously includes the Natural numbers set and also the Integer numbers set.

For instance: $2=\frac{2}{1}$ or $-15=\frac{-15}{1}$