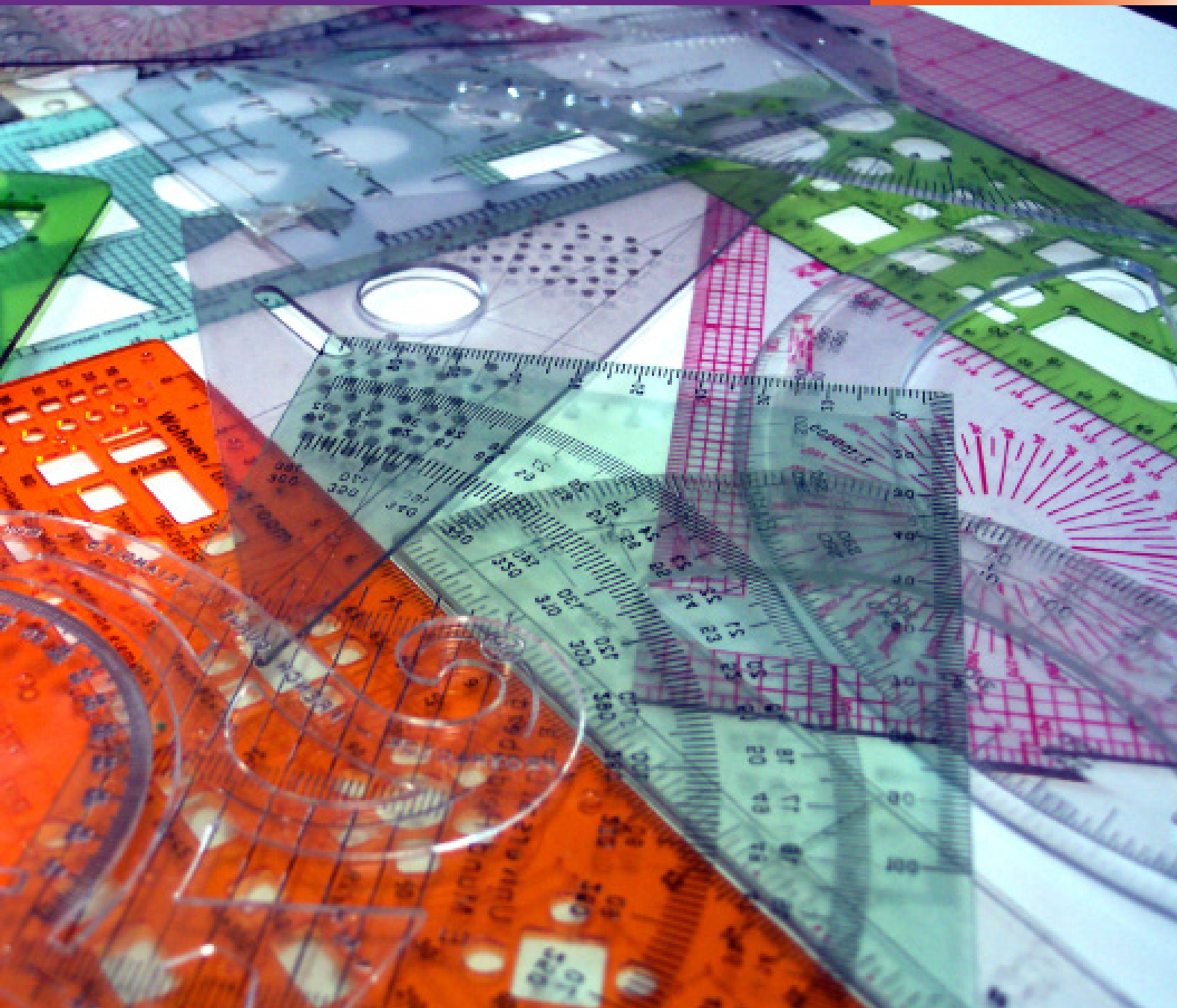




Rosario Carrasco Torres

MATHEMATICS

2 ESO



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Rosario Carrasco Torres



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Autora: Rosario Carrasco Torres

Maquetación: Daniela Vasilache y Rosario Carrasco Torres

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Avda. Jacarandas nº 2, loft 327 - 46100 Burjassot - València

Tel: 960 624 309 - 963 768 542 - 610 900 111

E-Mail: educaliaeditorial@e-ducalia.com

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PREFACIO

“La esencia de las Matemáticas no consiste en complicar lo que es simple, sino en simplificar lo que es complicado”.

Stanley Gudder

La enseñanza de las Matemáticas en la Educación Secundaria, lo sabemos bien los que nos dedicamos a ello, puede llegar a ser una tarea difícil y más aún si se realiza en una lengua que no es la lengua materna del alumnado.

Este libro surge precisamente de mi interés por elaborar un material sencillo y atractivo para el estudiante y que al mismo tiempo sea fiel al currículo de esta etapa. El hecho es que facilita muchísimo la labor del profesorado pues se trata de un compendio de contenidos, actividades y problemas que puede ser utilizado perfectamente como libro de texto, lo que es una gran ventaja como he podido comprobar personalmente en mi labor como profesora.

El libro está estructurado en unidades didácticas. Cada una de ellas consta de:

- Índice detallado.
- “Key Vocabulary”, es decir, el **vocabulario técnico de Matemáticas en inglés** que necesita el alumnado específicamente para cada unidad.
- Esquema introductorio, que informa de lo que se va a estudiar en la unidad correspondiente.
- **Contenidos**, que es la parte esencial de la misma y que está plagada de ejemplos, casos particulares, diagramas, gráficos... todo ello elaborado cuidando el detalle y el color.
- Una **lista de páginas web**, en inglés, donde los alumnos pueden practicar, experimentar e incluso aprender interactuando, los contenidos de la unidad.
- Una **tabla** donde se recoge el vocabulario nuevo aprendido y que el alumno debe completar con la fonética de cada palabra y su significado.
- Y por último se incluyen una **amplia colección de actividades, ejercicios y problemas de cada unidad didáctica**.

Se trata de un material adaptable a diversas metodologías pues está elaborado teniendo en cuenta los principios de la metodología AICLE (CLIL) que ofrece la posibilidad de aprender los contenidos curriculares de la asignatura de Matemáticas a la vez que permite practicar la lengua inglesa aprendida en etapas previas incrementando su bagaje de vocabulario técnico específico de Matemáticas en Inglés.

El hecho de que esté íntegramente elaborado en inglés constituye una ventaja para la inmersión total del alumnado en la lengua inglesa.

El alumno asimila palabras, frases y vocabulario cotidiano de la lengua inglesa además de estructuras y vocabulario específico de la propia asignatura de Matemáticas.

Es más, como es el propio alumno el que completa la tabla fonética de cada unidad, va interiorizándola sin apenas darse cuenta; tabla que puede ampliar con los términos que cada uno decida individualmente. Se trata en definitiva de un material que el alumno mismo ayuda a elaborar según sus necesidades particulares y que al final del año escolar le habrá servido para confeccionar su propia lista de vocabulario específico.

Otra ventaja de estos materiales es que por la **sencillez y concreción** con que están definidos los conceptos en el libro, se facilita el aprendizaje y se produce un impacto en la conceptualización, es decir, el alumno llega a ser capaz de pensar directamente en lo que se dice aunque esté expresado en otra lengua centrándose en los contenidos curriculares de la materia. Este aspecto ayuda a ampliar su mapa conceptual del pensamiento y a desarrollar en mayor medida sus competencias.

Y si alguna cosa más hubiera que destacar, personalmente destacaría la **motivación** que se logra en el alumnado al trabajar con este material. Por su estructura práctica, el colorido elegido al detalle para hacerlo atractivo, las listas de recursos web que se facilitan, porque las actividades y problemas planteados se ajustan a los contenidos y al nivel con propiedad, o por todo ello unido, se produce un efecto participativo y motivador que de otra forma es difícil alcanzar en la actualidad.

Por último decir que este libro está escrito con la ilusión de compartir y transmitir dos de mis grandes pasiones, la lengua inglesa y las Matemáticas, de la manera más sencilla posible.

ROSARIO CARRASCO TORRES

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3	Three	Third (3 rd)	23	Twenty-three	Twenty-third (23 rd)		
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17	Seventeen	Seventeenth (17 th)	100	One hundred	Hundredth		
18	Eighteen	Eighteenth (18 th)	1000	One thousand	Thousandth		
19	Nineteen	Nineteenth (19 th)	100000	One hundred thousand	Hundred thousandth		
20	Twenty	Twentieth (20 th)	1000000	One million	Millionth		

REMEMBER THAT:

- In decimal numbers we use a **comma** and in Britain they use a point.
- The figure 0 is usually called **nought** before a comma and **oh** after the comma. After the comma every digit is said **separately**.
Example: “0,30014 ≡ nought comma three oh oh one four”
- In team games, zero scores are usually called **nil**.
Example: “Spain two France nil”
- For the number **0** or for the temperatures we say **zero**.
Example: “-5° C ≡ five degrees below zero”
- “**and**” is used after any hundred digit of a number.
Examples: “214 ≡ two hundred and fourteen”; “502 ≡ five hundred and two”.
- Numbers are normally written in singular but you can use plural with hundred, thousand, million... if they are followed by “of”.
Examples: “Thousands of years ago...” ; “There are hundreds of insects...”
- **Fractions:** there are several ways to say a fraction.
Examples: $2/3 \rightarrow \begin{cases} \text{two over three} \\ \text{two thirds} \\ \text{two divided by 3} \end{cases}$ $9/7 \rightarrow \begin{cases} \text{nine over seven} \\ \text{nine sevenths} \\ \text{nine divided by seven} \end{cases}$

The particular case of 2 as denominator: $\frac{1}{2} \rightarrow$ one half / a half

UNIT 1: “INTEGERS”

1. Integer numbers
 - 1.1. Order and representation
 - 1.2. Absolute value
 - 1.3. Opposite number
2. Operations
 - 2.1. Addition and subtraction
 - 2.2. Multiplication and division
 - 2.3. Powers
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4. Order (hierarchy) of the operations (Bidmas)
5. Divisibility
 - 5.1. Rules of divisibility
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 - 5.3. G.C.D. (Greatest Common Divisor)
 - 5.4. L.C.M. (Lowest Common Multiple)

KEY VOCABULARY:

Positive number
 Plus
 Negative number
 Minus
 Integer (or whole number)
 Order
 Greater than
 Smaller than
 Absolute value
 Opposite
 Addition
 Subtraction
 To multiply
 To divide
 Factor of
 Prime number
 Power
 Base
 Exponent
 Even
 Odd
 Parenthesis
 Brackets
 Square root
 Radical
 Radicand
 Indices
 Hierarchy
 Divisible by
 Divisor of
 Remainder
 Composite number
 G.C.D. or G.C.F.
 L.C.M.

In this unit you will learn how to:

- *Order Integer numbers*
- *Operate with integer numbers*
- *Calculate the lowest common multiple and the highest common factor of some numbers*
- *Apply the properties of powers*
- *Express roots and powers*

1. Integer numbers

There are times when we can't express or solve some situations with the natural numbers (\mathbb{N}) and we need to enlarge that set to the integer numbers set (\mathbb{Z}).

The set of the integers is formed by:

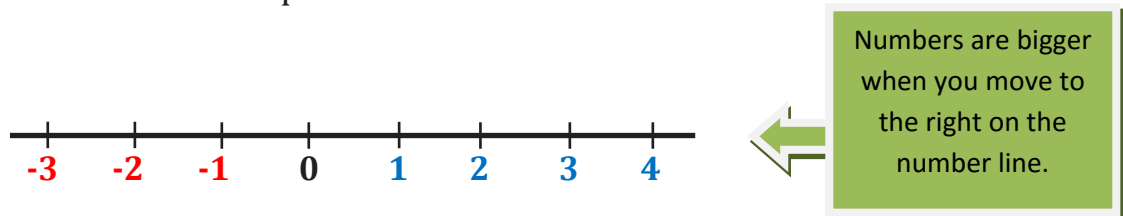
- positive integers: + 1, + 2, + 3 ...
- zero: 0
- negative integers: - 1, - 2, - 3 ...

plus (arrow pointing to positive integers)
minus (arrow pointing to negative integers)

REMEMBER THAT:
 $+ 2 = 2$

1.1. Order and representation

The integer numbers can be represented in the number line like that:



We can use symbols $<$ and $>$ to express which is the order relation between two numbers. For instance we can write: $-2 < -1$ $7 > -5$ $0 < 9$ $-6 > -8$

"-2 is smaller than -1"

"7 is greater than -5"

1.2. Absolute value

The absolute value of a number x is the number we obtain when we eliminate the negative sign of the number. It can be written $|x|$.

Examples: $|2| = 2$ $|-6| = 6$

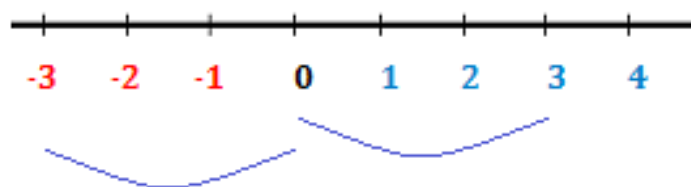
We can say that: the absolute value of -8 is 8.

1.3. Opposite number

The opposite of a number x is $-x$. It means that we must change the sign of the number. They both have the same absolute value but their signs are different. We can also represent $Op(x) = -x$.

Examples: $Op(11) = -11$ $Op(-3) = 3$

You can notice that two opposite numbers are at equal distance to the number zero in the number line.



2. Operations

2.1. Addition and subtraction

Addition:

- a) If the numbers have the same sign then add the absolute values of the numbers and write the sign they have. **Example:** $(-3) + (-5) = -8$
- b) If the numbers have different sign then subtract the absolute values of the numbers and write the sign of the one that has the highest absolute value. **Example:** $(-3) + (+5) = +2$

Subtraction:

To subtract integer numbers you must add the minuend to the opposite of the subtrahend. **Example:** $(-8) - (+6) = -14$

Remember the rule of the signs:

$$\begin{aligned} (+) \cdot (+) &= + & (+) \cdot (-) &= - \\ (-) \cdot (-) &= + & (-) \cdot (+) &= - \end{aligned}$$

2.2. Multiplication and division

To multiply and divide we must write the sign of the result following the rule of the signs and then we must do the operation as always.

Examples: $(-7) \cdot (+3) = -21$
 $(-8) : (-4) = +2$

THE SAME RULE FOR DIVISIONS

2.3. Powers

A power is the shortest way to express a product of a number by itself repeatedly.

$$a^n = a \cdot a \cdot a \dots \cdot a$$

(n times)

a is called the **base**
n is called the **exponent**

Example: 2^4 This expression can be read as: "Two raised to the fourth power"

5^2 can be read five squared
 7^3 can be read seven cubed

- "Two to the fourth power"
- "Two to the fourth"
- "Two to the power of four"

The most used

2.3.1. Expressions

SIGNS OF POWERS

* **EVEN** exponent \rightarrow **positive power** **Example:** $(+2)^4 = 16$
Example: $(-2)^4 = 16$

* **ODD** exponent:
 a) **IF THE BASE IS POSITIVE** \rightarrow **positive power** **Example:** $(+2)^3 = 8$
 b) **IF THE BASE IS NEGATIVE** \rightarrow **negative power** **Example:** $(-2)^3 = -8$

Take a look:

$$(+2) \cdot (+2) \cdot (+2) = 2 \cdot 2 \cdot 2 = 2^3$$

$$(-7) \cdot (-7) \cdot (-7) \cdot (-7) = (-7)^4 = 7^4$$

Surprisingly: $-7^4 \neq 7^4$

POWERS OF TEN

If we work with big numbers or very small numbers it is preferable to use powers of ten as in the examples:

$$876\,000\,000\,000\,000 = 876 \cdot 10^{12}$$

$$0,000\,005 = 5 \cdot 10^{-6}$$

2.3.2. Properties of powers

- The product of powers with the same base is another power with the same base and the exponent is the addition of the exponents.

$$a^n \cdot a^m = a^{n+m}$$

$$2^3 \cdot 2^4 = 2^7$$

- The quotient of powers with the same base is another power with the same base and the exponent is the subtraction of the exponents.

$$a^n : a^m = a^{n-m}$$

$$2^5 : 2^2 = 2^3$$

- The power of a power is another power with the same base and the exponent is the product of the exponents.

$$(a^n)^m = a^{n \cdot m}$$

$$(2^5)^3 = 2^{15}$$

- The power of a product (quotient) is the product (quotient) of the powers of the numerator and the denominator.

$$(a \cdot b)^n = a^n \cdot b^n$$

$$(7 \cdot 5)^2 = 7^2 \cdot 5^2$$

$$(a : b)^n = a^n : b^n$$

$$(15 : 5)^2 = 15^2 : 5^2$$

REMEMBER THAT:

$$a^0 = 1$$

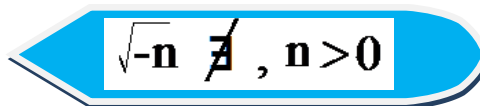
$$a^1 = a$$

3. Square roots

The **square root** of a number **a** is another number **b** that multiplied by itself gives the number **a**. It means that: $\sqrt{a} = b \leftrightarrow b^2 = a$

The symbol $\sqrt{\quad}$ is called **radical symbol** and the number **a** is called **radicand**. We also can use **radical** to refer to \sqrt{a} .

Example: $\sqrt{9} = 3$ since $3^2 = 9$



4. Order or hierarchy of the operations (Bidmas)

The hierarchy of the operations between integer numbers is usually remembered thanks to a mnemonic rule called: **BIDMAS**

Brackets - **Indices** - **Divisions** - **Multiplications** - **Additions** - **Subtractions**

The order of operations is one of the golden rules.

Take a look:

$$8 - 4 \cdot 6 = -16$$

Example: $[-11 - 5 \cdot (-3)^2] : \sqrt{64} = [-11 - 45] : 8 = (-56) : 8 = -7$

5. Divisibility

➤ A division is **exact** when the remainder is 0. Example:

In that case we can say that:

- 10 is a **multiple** of 5
- 5 is a **divisor** of 10
- 10 is **divisible** by 5
- 5 is a **factor** of 10

➤ A division is **not exact** when the remainder is not 0.

Generally speaking we can say:

a is multiple of b
b is a divisor of a

The set of multiples of a number **a** can be expressed by \dot{a} .

Example: $\dot{5} = \{ \dots, -15, -10, -5, 0, 5, 10, 15, 25, \dots \}$

➤ If a number can be divided only by 1 and by itself then it is a **prime number**.

Examples: 7, 2, 11, 17, 13, -11...

➤ Integer numbers that have more than two divisors are called **composite numbers**. Examples: 4, 16, -25,...

5.1. Rules of divisibility

RULES OF DIVISIBILITY	The number is divisible by:
The last digit is 0 or even	2
The sum of the digits is divisible by 3	3
The last digit is 0 or 5	5
The sum of the digits is divisible by 9	9
The last digit is 0	10
The (sum of the odd positioned digits) - (sum of the even positioned digits) is divisible by 0 or 11. Example: 34871903 $3 + 8 + 1 + 0 = 12$ $4 + 7 + 9 + 3 = 23$ $23 - 12 = 11$ Then the number is divisible by 11	11
TAKE A LOOK: If a number is divisible by two different prime numbers, then it is divisible by the product of those two numbers. Since 12, is divisible by both 2 and 3, it is also divisible by 6.	

5.2. Prime factorization

The process of finding the prime numbers that divide exactly an integer number is called **prime factorization** or **integer factorization**.

$$\begin{array}{r|l}
 24 & 2 \\
 12 & 2 \\
 6 & 2 \\
 3 & 3 \\
 1 &
 \end{array}$$

$$24 = 2^3 \cdot 3$$

$$\begin{array}{r|l}
 42 & 2 \\
 21 & 3 \\
 7 & 7 \\
 1 &
 \end{array}$$

$$-42 = (-1) \cdot 2 \cdot 3 \cdot 7$$

When the number is negative we add - 1 to the prime factorization.

5.3. G.C.D. (Greatest Common Divisor or Factor G.C.F.)

The Greatest Common divisor of several numbers is the largest of their common divisors.

To calculate the G.C.D.:

First step: calculate the prime factorization of the numbers

Second step: choose the **common factors** raised to the **smallest exponent**

TAKE A LOOK:

$$\text{G.C.D. } (15, -27) = \text{G.C.D. } (15, 27) = 3$$

Example:

Calculate the G.C.D. (45, 24, 12)

- $45 = 3^2 \cdot 5$
- $24 = 2^3 \cdot 3$
- $12 = 2^2 \cdot 3$
- G.C.D. (45, 24, 12) = 3

Special case: G.C.D. (a, b) = 1 if a and b don't have any common divisor.

Example: G.C.D. (25, 12) = 1

5.4. L.C.M. (Lowest Common Multiple)

The Lowest Common Multiple of several numbers is the smallest of their common multiples.

To calculate the L.C.M.:

First step: calculate the prime factorization of the numbers

Second step: choose the **common and not common** factors raised to the **biggest exponent**

Example:

Calculate the L.C.M. (45, 24, 12):
 L.C.M. (45, 24, 12) = $2^3 \cdot 3^2 \cdot 5 = 360$

TAKE A LOOK:

L.C.M. (15, - 27) = L.C.M. (15, 27) = 135

PRACTISING INTEGERS USING WEBSITES

- ⇒ <http://www.bbc.co.uk/schools/gcsebitesize/maths/number>
- ⇒ http://amby.com/educate/math/integ_x1.html
- ⇒ <http://mathleague.com/help/integers/integers.htm>

VOCABULARY	phonetics	meaning
Positive number		
Plus		
Negative number		
Minus		
Integer		
Whole number		
Order		
Greater than		
Smaller than		
Absolute value		
Opposite		
Addition		
Subtraction		
To multiply		
Factor of		
Prime number		
Power		
Base		
Exponent		
Even		
Odd		
Parenthesis		
Brackets		
Square root		
Radical		
Radicand		
Indices		
Hierarchy		
Divisible by		
Divisor of		
Remainder		
Composite number		



EXERCISES

1. Order the following numbers from the lowest to the highest:

$-3, 4, -1, -25, 5, -6, 0, 1, -9$ and -7 .

2. Insert the correct symbol into each box:

$4 \square -3$; $-2 \square -1$; $-1 \square 0$; $1 \square 0$;
 $+3 \square -3$; $-6 \square -16$; $0 \square -4$; $-7 \square -8$



3. What are the integer numbers between -6 and 4 ?

4. Solve the following calculations involving absolute values:

a) $|-12| =$ b) $|-1| - |-4| =$ c) $-|+8| + |-3| =$ d) $|-9| - |+9| =$ e) $-2 \cdot |-6| =$

5. Calculate the opposite number of each one of the following numbers:

$7, -5, 11, -9, 4, -1, 13, 56$ and -100

6. Work out the following calculations:

a) $(+3) - (-4) =$ b) $-(-2) + (-4) =$ c) $(+7) + (-8) =$
 d) $0 - (-6) - 7 =$ e) $(5) + (-3) =$ f) $(+4) + (-4) =$
 g) $-3 + 4 - 2 + 7 =$ h) $7 - 9 + 6 - 1 - 3 =$ i) $(-4) - (+6) + (-4) - (-1) =$
 j) $-(-1) - (+8) + (+5) =$ k) $+(+2) - (-9) - (+3) =$ l) $-[-(+3) - (-1)](-11) =$
 m) $2[-3+4] - 3[-7+8] =$ n) $-7 - 3[5-8] - 6 : 3 =$ o) $(-3) \cdot (-4) : (+2) =$
 p) $(-21) : (+7) \cdot (-1) =$ q) $15 : (-3) + (-4) =$ r) $(8-4) : 2 \cdot 7 - 9 + 4 \cdot 5 =$
 s) $(12 : 4 - 3 \cdot 4) : (-3) =$ t) $-[-(-5) + 7] : 4 =$ u) $(-25) : (-5) - [-14 : 2] =$
 v) $4 - 5 \cdot (-7) + (-2) \cdot 3 =$ w) $1 - 2[4 - 3 \cdot 6] - 1 =$

7. Calculate:

a) $(-3)^2 =$ b) $-(-2)^3 =$ c) $(-3)^0 =$ d) $-2^4 =$ e) $(-2)^4 =$ f) $-(-4)^2 =$

8. Simplify the expressions and give the result as a power if possible:

a) $(2^6 : 2^3) \cdot 2 =$ b) $(3^7 : 3^2) : 3 =$ c) $(5^3 \cdot 5^2 \cdot 5^7) \cdot 5^4 : 5^0 =$ d) $[(-2)^4 : (-2)^2] : 2 =$
 e) $[(-4)^3 \cdot (-4)^4] : (-4)^5 =$ f) $7^{20} : [(7^3)^6] : 7 =$ g) $[(-5)^3]^5 : (-5)^7 =$
 h) $3^4 \cdot 9^2 =$ i) $25^3 : 5^2 =$ j) $49^4 : 343^2 =$ k) $2^6 \cdot 8^3 \cdot 4^2 =$

9. Solve the following mixed operations involving powers and roots:

a) $- (+2)^3 + (-4)^2 =$ b) $(-5)^2 - (\sqrt{25})^2 =$ c) $(3^2 : 3^2)^4 =$
 d) $(-6)^4 : 6^2 =$ e) $1 - [(-5)^2 - 7 \cdot \sqrt{9}] =$ f) $(-3)^2 - (-2^2)^2 + [10 : (-5)] =$
 g) $\sqrt{49} - (-2) : (+2) - 5 \cdot 3 - (-3^2) =$ h) $(-2) \cdot (-2) \cdot (-2)^5 \cdot (-2) - (-1) \cdot (-1)^3 =$
 i) $(-3)^4 \cdot 3^2 \cdot 3 \cdot 3^6 - \sqrt{36} - 2^2 =$

10. Write down 7 multiples of 3.

11. Calculate the divisors of 36 and find out which are prime numbers.

12. Find out the divisors of 42 that are between 7 and 30.

13. Factorize the following numbers: 1122, -1925, 121, 3700 and -875.
14. Find the G.C.D. of the following set of numbers:
- | | | |
|-------------------|------------------|------------------|
| a) 27 and 90 | b) 21 and 56 | c) -15 and 75 |
| d) 17 and 48 | e) 121 and 22 | f) 8, 14 and 12 |
| g) -64, 21 and 28 | h) 12, 18 and 90 | i) 30, -9 and 54 |
15. Calculate the L.C.M. of the sets of numbers above.
16. Name two numbers whose G.C.F. is 6.
17. Name two numbers whose G.C.F. is 1.
18. Name two numbers whose L.C.M. is 60.
19. True or false:
- | | |
|---|-----------------------|
| a) 5 is a divisor of 125. | b) 6 is multiple of 2 |
| c) 12 is a multiple of 3 and a divisor of 25. | |

WORD PROBLEMS

20. Anna has 15 € for the weekend. She pays 9€ for some drinks for her friends. And her friends give Anna back 6 € they owed to her. How much money does she have now?
21. The temperature at eight o'clock in the morning was 12°C. Two hours later it was 4 degrees higher but suddenly, it has decreased 6 degrees because of a winter storm. What is the temperature now?
22. The lift of a hotel doesn't work properly. Carmen doesn't know that so she gets in the elevator on the 2nd floor and presses the button to go to the hall of the hotel. But the lift, instead of going down, goes up seven floors, then stops, then goes down five floors, then four more and finally the lift stops and Carmen gets out. Is she finally in the hall as she wanted?
23. A baker makes biscuits every two days, cakes once a week and chocolate cookies every three days. How often is he going to be very busy making the three kinds of sweets the same day?
24. A shop assistant wants to cut three ropes that measure 9m, 12m, and 15m into equal pieces as big as possible without wasting any piece of the ropes. What will be the length of the pieces?
25. Richard has candies of three different flavours: 120 of mint, 80 of strawberry and 150 of lemon to share on the day of his birthday at school. What is the maximum number of individual bags he can make with the same number of candies of every flavour?