# MATHEMATICS

# 2º ESO

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educàlia
editorial

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# LESSON 1: DIVISIBILITY AND INTEGERS

***Keywords***

**Multiple** Prime Composite LOWEST COMMON MULTIPLE
Factor **Highest Common Factor** BODMAS
Powers BASE EXPONENT Square Root

## 1. FACTORS AND MULTIPLES

***Factors*** of a natural number are numbers that divide exactly into it (with zero as remainder).

*Example*: The factors of 12 are 1, 2, 3, 4, 6 and 12. (If you divide 12 by them you get remainder zero).

***Multiples*** of a natural number are numbers that you get multiplying it.

*Example*: 3, 6, 9, 12, … are multiples of 3 because

$$3 = 3 \cdot 1$$
$$6 = 3 \cdot 2$$
$$9 = 3 \cdot 3$$
$$12 = 3 \cdot 4$$

*Question*: Is 46 a multiple of 5?

*Observe*:

1. If 3 is a factor of 12, 12 is a multiple of 3, and we say that 12 is divisible by 3.

$$\begin{array}{r|l} a & b \\ \hline 0 & c \end{array}$$

- $a$ is divisible by $b$
- $b$ is a factor of $a$
- $c$ is a factor of $a$
- $a$ is multiple of $b$

2. a is multiple of a.
3. a is a factor of a.
4. 1 is a factor of all numbers.
5. A number has infinite multiples.
6. If you add two multiples of n, the result is a multiple of n.

*Example*:

$$\left.\begin{array}{l}\text{6 is a multiple of 3}\\ \text{9 is a multiple of 3}\end{array}\right\} \Longrightarrow \text{6 + 9 = 15 is a multiple of a 3}$$

## 2. DIVISIBILITY RULES

### 2.1. Divisibility by 2

A number is divisible by 2 if it ends in an even digit (0, 2, 4, 6 or 8).
*Examples*: 2, 4, 6, 8754, 986432… are divisible by 2.

L E S S O N 1

### 2.2. Divisibility by 3

How to know if a number is divisible by 3:
1. Add up all the digits in the number.
2. If the result is multiple of 3, the number is divisible by 3.

*Example*: 5415 is divisible by 3 ($5 + 4 + 1 + 5 = 15$ multiple of 3)

### 2.3. Divisibility by 5

Numbers ending in a 5 or a 0 are divisible by 5.

*Example*: 876435 is divisible by 5 (it ends in a 5).

### 2.4. Divisibility by 10

Numbers ending in a 0 are divisible by 10.

*Example*: 8754330 is divisible by 10.

## 3. PRIME AND COMPOSITE NUMBERS

- A ***prime number*** is a natural number that has exactly two factors: 1 and itself.
  *Note*: 1 is not considered to be a prime number.

- A ***composite number*** is a natural number that has more than two factors. It is a product of prime numbers.

*Exercise*: The "Sieve of Erathostenes" is a procedure to find the first prime numbers. Use it to find out the prime numbers less than 50.

## 4. PRIME FACTOR DESCOMPOSITION OF A NUMBER

The ***factoring of a number*** means writing it as a product of prime factors (using powers).

How to factor a number:
1. Try to divide the number by the first prime number, which is two. If you get remainder zero try to divide by two again, if not, divide by the following prime number, 3.
2. Repeat step 1 with the following prime numbers until the final quotient is 1.

*Example*: Factor 72.

$$\begin{array}{r|l} 72 & 2 \\ 36 & 2 \\ 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & 1 \end{array}$$

So, we have the factoring:
$$72 = 2^3 \cdot 3^2$$

*Note*: Factors of a number are obtained by multiplying its prime factors.

## 5. LOWEST COMMON MULTIPLE (LCM)

It is the smallest common multiple of two or more numbers (you can divide the LCM of two numbers by both of them exactly).

How to calculate LCM:
- Factor the numbers.
- Choose **non-common and common** factors with the **highest exponents**.
- The LCM is the product of those powers.

*Example*: Find the LCM of 36 and 40.

$$\begin{array}{r|l} 36 & 2 \\ 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array} \qquad \begin{array}{r|l} 40 & 2 \\ 20 & 2 \\ 10 & 2 \\ 5 & 5 \\ 1 & \end{array}$$

$36 = 2^2 \cdot 3^2$ $\qquad$ $40 = 2^3 \cdot 5$

$$LCM\ (36, 40) = 2^3 \cdot 3^2 \cdot 5 = 360$$

## 6. HIGHEST COMMON FACTOR (HCF)

It is the largest number that is a factor of all of them at the same time.

How to calculate the HCF:
- Factor the numbers.
- Choose **common** factors with the **lowest exponents**.
- The HCF is the product of those powers.

*Example*: Find the HCF of 36 and 40.

$$\begin{array}{r|l} 36 & 2 \\ 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array} \qquad \begin{array}{r|l} 40 & 2 \\ 20 & 2 \\ 10 & 2 \\ 5 & 5 \\ 1 & \end{array}$$

$36 = 2^2 \cdot 3^2$ $\qquad$ $40 = 2^3 \cdot 5$

$$HCF\ (36, 40) = 2^2 = 4$$

## 7. ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION WITH INTEGERS

### 7.1. Addition

➢ Integers with the **same sign**: you keep the sign and add the numbers.

*Examples*:

a) (+3) + (+5) = (+8) $\qquad$ b) (-4) + (-7) = (-11)

LESSON 1

➢ Integers with **different sign**: you choose the sign of the highest number and subtract them.

*Examples*:

a) (+9) + (- 4) = (+5)

b) (+4) + (- 15) = (- 11)

### 7.2. Subtraction

To subtract integers change the sign of the number that is to be subtracted, then you do it like additions.

*Examples*:

a) (-3) - (+7) = -3 -7 = -10

b) 5 - (-8) = 5 + 8 = 13

### 7.3. Multiplication

- First you have to multiply signs.

$$(+) \cdot (+) = (+)$$
$$(+) \cdot (-) = (-)$$
$$(-) \cdot (+) = (-)$$
$$(-) \cdot (-) = (+)$$

- Then, you multiply numbers.

*Examples*:

a) (-3) · (+6) = (-18)

b) (-5) · (-9) = +45

### 7.4. Division

- First you have to divide signs.

$$(+) : (+) = (+)$$
$$(+) : (-) = (-)$$
$$(-) : (+) = (-)$$
$$(-) : (-) = (+)$$

- Then, you divide numbers.

*Examples*:

a) (-14) : (+7) = -2

b) (-55) : (-5) = +11

## 8. POWERS OF INTEGERS

$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ Two to the power of 5 or two to the fifth

Base $\longrightarrow 2^5 \longleftarrow$ Exponent

*Properties*:

1. $a^0 = 1$
2. $a^1 = a$

3. The power of a product is the product of the powers.
   *Example*: $(2 \cdot 3)^4 = 2^4 \cdot 3^4$

$$(a \cdot b)^n = a^n \cdot b^n$$

4. The power of a quotient is the quotient of the powers.
   *Example*: $(2 : 3)^4 = 2^4 : 3^4$

$$(a : b)^n = a^n : b^n$$

5. When multiplying powers *of the same base*, you keep the same base and add the exponents.

$a^n \cdot a^m = a^{n+m}$

*Example*: $5^3 \cdot 5^4 = 5^7$

6. When dividing powers *of the same base*, you keep the same base and subtract the exponents.

$a^n : a^m = a^{n-m}$

*Example*: $7^8 : 7^5 = 7^3$

7. When powering a power, you keep the base and multiply the exponents.

$(a^n)^m = a^{n \cdot m}$

*Example*: $(4^3)^2 = 4^6$

8. If the base is a negative number there are two options:
   - If the exponent is an even number, the result is a positive number.
   - If the exponent is an odd number, the result is a negative number.

*Examples*:

a) $(+7)^3 = 7 \cdot 7 \cdot 7 = 343$ b) $(-5)^2 = +25$ c) $(-3)^3 = -27$

## 9. SQUARE ROOT OF INTEGERS

$\sqrt{a} = b \Longrightarrow b^2 = a$ ($a$ radicand, $b$ root)

The opposite of squaring a number is calculating its square root.

*Example*: $\sqrt{9} = 3$ The square root of nine equals 3

✓ The square root of a negative number doesn't exist.
✓ The square root of a positive number has two solutions, one positive and one negative.
*Example*:

$$\sqrt{16} = \pm 4 \quad \text{because} \quad (+4)^2 = 16 \quad \text{and} \quad (-4)^2 = 16$$

## 10. *n*-th ROOTS

$$1.532323232... = 1.5\overline{32} = \frac{1532-15}{990} = \frac{1517}{990}$$

$\sqrt[n]{a} = b \Rightarrow b^n = a$

We read $\sqrt[n]{a}$ as "the $n$-th root of $a$" or "the root of index $n$ of $a$

*Examples*: a) $\sqrt[3]{8} = 2$ because $2^3 = 8$

b) $\sqrt[3]{-8} = -2$ because $(-2)^3 = -8$

c) $\sqrt[4]{16} = \pm 2$ because $2^4 = 16$ and $(-2)^4 = 16$

d) $\sqrt[4]{-16}$ does not exist.

# WORKSHEET

***Exercise 1.*** Each of these calculations has an answer which is one of the numbers of the list: 12, -12, -3, -4, 3, 8.
Select the correct number for each answer:

a) $3\cdot(-4)=.......$

b) $-16\div(-2)=.......$

c) $8-(-4)=.......$

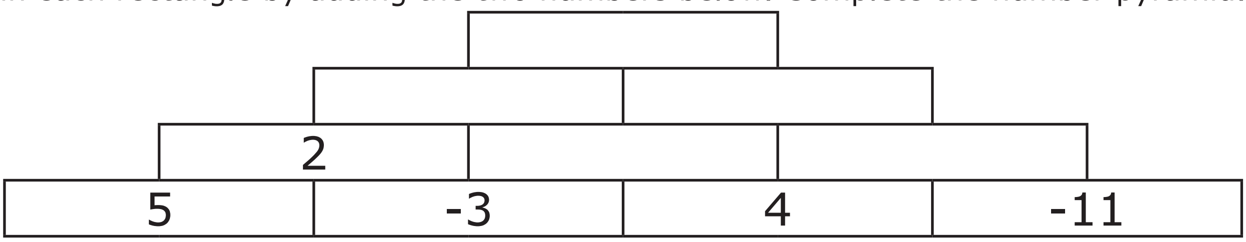
d) $4+(-7)=.......$

e) $-9\div(-3)=.......$

f) $-7-(-3)=.......$

***Exercise 2.*** The temperature at midday is -2ºC. By midnight it has dropped 10 degrees. What is the temperature at midnight?

***Exercise 3.*** Here are some numbers in a number pyramid. You calculate the number in each rectangle by adding the two numbers below. Complete the number pyramid.




***Exercise 4.*** Here are some signs: $\boxed{+}$ $\boxed{-}$ $\boxed{\times}$ $\boxed{\div}$

Insert the correct sign to make each calculation correct:

a) $-12\,...\,(-3)=4$

b) $3\,...\,(-3)=-9$

c) $-7\,...\,(-3)=-4$

d) $12\,...\,(-5)=7$

***Exercise 5.*** Here is a list of numbers: -9, -5, -1, 0, 3, 2, 5

Choose the number from the list to make each sentence correct:

a) $3-...=4$

b) $5\cdot...=-25$

c) $-9+...=-4$

d) $2-...=7$

e) $-9\div...=-3$

**Exercise 6.** A farmer can make groups of 5, 6 or 9 hens without leaving any of his hens out. There are less than 100 hens in this farm. How many hens are there in the farm?

**Exercise 7.** A cook in Iowa made a huge pizza on June 19th 2005 and he entered the Guinness book of records. It was a rectangular pizza which was 44 m. long and 33 m. wide.

We are going to divide this pizza into square portions which are as big as possible without throwing away any pizza.

a) How long should each side of the pizza be?

b) How many portions are we going to get?

**Exercise 8.** Two athletes A and B started running from the starting line of a stadium at the same time. Both of them ran at a constant speed and it took them 84 seconds and 96 seconds to finish a lap.

a) How often did they cross the starting line together?

b) How many laps had each of them completed when they crossed the starting line together for the first time after the race had started?

**Exercise 9.** We want to cut two wires which are 30 m and 66 m long into pieces of the same length that are as big as possible and without wasting any wire. How long is each piece?

# LESSON 2: DECIMAL SYSTEM AND SEXAGESIMAL SYSTEM

**Keywords**

Exact decimal    Repeating decimal    Point    PERIOD
Irrational number    ROUNDING    NUMBER LINE
Sexagesimal system    digit    DEGREES

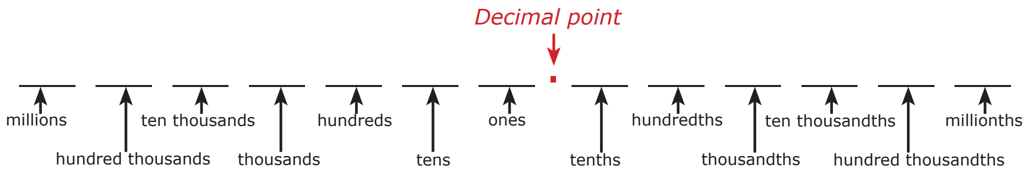
## 1. DECIMAL SYSTEM

A ***Decimal number*** has two parts separated by a decimal point.

*Example*: 13 is the whole part; 147 is the decimal part
We read 13.147 as "thirteen, point, one, four, seven".

Nowadays, we often use decimal system. For example, we read 5.24 € as "five point twenty four euros" or "five euros and twenty four cents" or we read 5.36 m as "five point thirty six metres".

Decimal system is a base 10 system: on this system every unit is divided into 10 equal parts to get the subunit. So, each digit has a value which is 10 times larger than the following digit on its right.




## 2. TYPES OF DECIMAL NUMBERS

There are three types of decimal numbers:

- **Exact Decimals (or Terminating Decimals)**. -There is a limited quantity of digits in the decimal part.

*Example*: 13.121

- **Repeating Decimals (or Recurring Decimals)**. -There is an unlimited quantity of digits in the decimal part and there is a group of digits which are repeated forever.

*Examples*: If you divide 1 by 3 you get 0.3333333...
If you divide 5 by 22 you get 0.2272727...

The group of repeated decimal digits is called **period**. We usually write $\frown$ over the period.

*Examples*: $0.3333333... = 0.\overset{\frown}{3}$ (the period is 3)

$0.2272727... = 0.2\overset{\frown}{27}$ (the period is 27)

- **Irrational Numbers**. There is an unlimited quantity of digits in the decimal part but there is not any period.

*Examples*: Calculating $\sqrt{2}$ we get 1.414213562... (it is an irrational number)

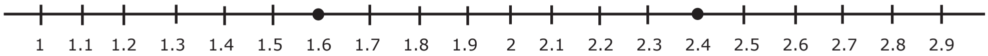
$\pi = 3.14159265358979323846 2643...$ (it is an irrational number)



## 3. DECIMAL NUMBERS ON THE NUMBER LINE

Every decimal number has a place on the number line between two integer numbers.

*Example*: We can represent the numbers 1.6 or 2.4 dividing the units into ten equal parts:




- Property: Between two decimal numbers there are infinite decimal numbers.

*Examples*:
Between 1.6 and 1.7 we can find, for example, 1.63 and 1.6<1.63<1.7
Between 1.63 and 1.64 we can find, for example, 1.637 and 1.63<1.637<1.64

## 4. ROUNDING DECIMAL NUMBERS

To round a number to any decimal place value we look at the digit to the right of the place we wish to round to and when the digit 5, 6, 7, 8, or 9, appears in that place, you must add one unit to the last digit; when the digit 0, 1, 2, 3, or 4 appears in that place, you must cut the number.

*Examples*:
1.38 rounded to the nearest tenth is 1.4
2.36321 rounded to the nearest hundredth is 2.36

## 5. SEXAGESIMAL SYSTEM

We frequently use sexagesimal system when we talk about angles and time. In sexagesimal system every unit is divided into 60 equal parts to get the sub-unit.

- Angles:
The unit is the degree (º). The subunits are minutes (') and seconds (").

One minute: $1' = \frac{1}{60}$ of a degree. That is, $60' = 1^{o}$

One second: $1'' = \frac{1}{60}$ of a minute. That is, $60'' = 1'$

*Example*: An angle "A" is expressed as $A = 43^{o}12'5''$

-<u>Time</u>:
The unit is the hour (h). The subunits are minutes (min) and seconds (s).

One minute: 1 min = $\frac{1}{60}$ of an hour. That is, 60 min = 1 h

One second: 1 s= $\frac{1}{60}$ of a minute. That is, 60 s = 1 min

*<u>Example</u>*: A period of time is expressed as 3 h 5 min 3 s.

## 6. OPERATIONS IN THE SEXAGESIMAL SYSTEM

### 6.1. Addition

We need to add degrees (or hours), minutes and seconds separately and then convert the seconds into minutes and the minutes into degrees (or hours) if we get more than 60 subunits.

*<u>Example</u>*: Add 45º 53' 40" + 12º 33' 35"
Adding separately we get 45º 53' 40" +12º 33' 35" = 57º 86' 75"
but 75" = 1' 15" so we add 1' and get 87' = 1º 27' we add 1º and finally the solution is 57º 86' 75" = 58º 27' 15"

### 6.2. Subtraction

We need to subtract degrees (or hours), minutes and seconds separately, if we do not have enough seconds or minutes we convert one degree (or hour) into minutes or a minute into seconds.

*<u>Example</u>*: Subtract 57 h 13 min 21 s and 12 h 43 min 5s
We write 57 h 13 min 21 s as 56 h 73 min 21s and then we can subtract.
So, the solution is 45 h 30 min 16 s

### 6.3. Multiplication by a natural number

We multiply degrees minutes and seconds separately and then convert the seconds into minutes and the minutes into hours (or degrees) when we get more than 60 subunits.

*<u>Example</u>*: $(3\,h\ 22\,\text{min}\ 25\ s)\cdot 4$

$$\begin{array}{ccc} 3\ h & 22\,\text{min} & 25\,s \\ \cdot 4 & \cdot 4 & \cdot 4 \\ \hline 12\ h & 88\,\text{min} & 100\,s \\ 13\ h & 89\,\text{min} & \boxed{1\,\text{min}}\,40\ s \\ & \boxed{1\ h}\,29\,\text{min} & \end{array}$$

So, the solution is 13 h 29 min 40 s

### 6.4. Division by a natural number

We divide the hours (or degrees), and the remainder is converted into minutes that must be added to the previous quantity of minutes that we had, divide the minutes and we repeat the same that we have done before. The remainder is in seconds.

*Example*:

$$
\begin{array}{lll|l}
46^{\circ} & 15' & 23'' & 4 \\ \cline{4-4}
2^{\circ}\cdot 60 = \underline{120'} & & & 11^{\circ}\ 33'\ 50'' \leftarrow \text{Quotient} \\
 & 135' & & \\
 & 3'\cdot 60 = \underline{180''} & & \\
 & & 203'' & \\
 & & 3'' \leftarrow \text{Remainder} &
\end{array}
$$

# WORKSHEET

**Exercise 1.** Here are some numbers in a number pyramid. The number in each rectangle is found by adding the two numbers below. Complete the number pyramid.

| | | | |
|---|---|---|---|
| | | | |
| | | | |
| 6.3 | 12.4 | | |
| 2.7 | 3.6 | | 8.1 |

**Exercise 2.** A piece of paper is 0.01 cm thick. A notebook has 120 sheets of paper. How thick is the notebook?

**Exercise 3.** Order these numbers from the lowest to the greatest:

3,0707 ; $3,\overset{\frown}{07}$ ; 3 ; 3,0007 ; 3,077 ; 3,07 ; $3,\overset{\frown}{7}$

**Exercise 4.** Decide whether each statement is true or false:

a) 2.73 rounded to the nearest tenth is 2.7 …………
b) 12.278 rounded to the nearest hundredth is 12.27 …………
c) 125.72987 rounded to the nearest thousandth is 125.73 …………
d) 3.789 rounded to the nearest thousandth is 3.79 …………

**Exercise 5.** The atomic mass of one atom of hydrogen is 1.00794 grams, the mass of one atom of sulfur is 32.065 grams, and the mass of one atom of oxygen is 15.994 grams. What is the weight of one molecule of sulfuric acid, which contains two hydrogen atoms, one sulfur atom, and four oxygen atoms?

**Exercise 6.** A jar contains 0.346 litres of water. Every hour, 0.106 litres are added to the jar and 0.055 litres evaporate. How much water is there in the jar after five hours?

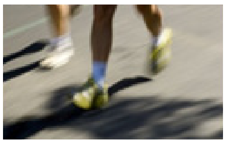
**Exercise 7.** Rufus makes $43.75 per day at his part-time job. If he wants to buy a laptop that costs $525, how many days must he work?

**Exercise 8.** Eva is on a diet and cannot eat more than 600 calories per meal. Her lunch yesterday consisted of 125 g. of bread, 140 g. of asparagus, 45 g. of cheese and an apple which weighed 130g. Check the following calory counter and decide if Eva followed her diet yesterday:

1g. bread: 3,3 calories
1g. asparagus: 0,32 calories
1g. cheese: 1,2 calories
1g. apple: 1,52 calories.

**Exercise 9.** Almost 40000 people ran the New York marathon which took place on 1st November 2009. Meb Keflezighi, from US, finished first. His time was 2 h 9 min 15 s. The first Spaniard, Francisco Ribera (2 h 26 min 48 s), came in 38th place.

a) How long was it between Meb's and Francisco's arrivals?
b) How long would it take an athlete who ran three times more slowly than Francisco Ribera?



**Exercise 10.** I spend 1 h 12 min going from Albacete to Peñas de San Pedro riding my bicycle. Running I need the double of the time and using my car I need the third.

a) How long does it take me to go from Albacete to Peñas de San Pedro running?
b) How long does it take me to go from Albacete to Peñas de San Pedro driving my car?