MATHEMATICS

1º ESO

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Juan Ignacio González Pérez

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INTRODUCTION: NUMBERS

You have already studied ***cardinal*** and ***ordinal*** numbers before, but let's revise them next.
You use ***cardinal numbers*** to count.
When objects are placed in order, we use ***ordinal numbers*** to tell their position.

NUMBER	CARDINAL	ORDINAL
0	zero	
1	one	first (1st)
2	two	second (2nd)
3	three	third (3rd)
4	four	fourth (4th)
5	five	fifth (5th)
6	six	sixth (6th)
7	seven	seventh
8	eight	eighth
9	nine	ninth
10	ten	tenth
11	eleven	eleventh
12	twelve	twelfth
13	thirteen	thirteenth
14	fourteen	fourteenth
15	fifteen	fifteenth
16	sixteen	sixteenth
17	seventeen	seventeenth
18	eighteen	eighteenth
19	nineteen	eighteenth
20	twenty	twentieth
21	twenty-one	twenty-first
22	twenty-two	twenty-second
23	twenty-three	twenty-third
24	twenty-four	twenty-fourth
25	twenty-five	twenty-fifth
26	twenty-six	twenty-sixth
27	twenty-seven	twenty-seventh
28	twenty-eight	twenty-eighth
29	twenty-nine	twenty-ninth
30	thirty	thirtieth
40	forty	fortieth
50	fifty	fiftieth
60	sixty	sixtieth
70	seventy	seventieth
80	eighty	eightieth
90	ninety	ninetieth
100	one hundred	hundredth
1.000	one thousand	thousandth
100.000	one hundred thousand	hundred thousandth
1.000.000	one million	millionth

Remember

- **Write a hyphen between the tens and units.**
  23: Twenty-three
- **Hundred AND ...**

  134: one hundred AND thirty-four
- **In English, the *POINT* is used for decimal numbers and the COMMA for thousand, million...**
  - **2,304 Two thousand three hundred and four.**
  - **5.30€ Five euros and thirty cents.**
- **We are in Spain, so we will use the *POINT* and the *COMMA* like this**
  - **2.304 Two thousand three hundred and four.**
  - **5,30€ Five euros and thirty cents.**

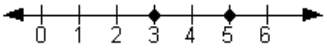
LESSON 1: NATURAL NUMBERS

Keywords

Natural Whole STRAIGHT LINE Round Addition
Add PLUS Subtraction Subtract Minus
MULTIPLICATION Times Factors System

1. WHAT ARE NATURAL NUMBERS?

- *Set*: Collection of objects, these objects are called *elements*.
- *Natural (or counting) Numbers*: 1, 2, 3, 4 …
- *Whole numbers*: 0, 1, 2, 3, 4 …
- *Straight line*: Used for representing numbers.



2. NUMBER SYSTEMS

Men used different systems to write numbers.

2.1. Prehistoric System

The prehistoric men used symbols like:

=20 =5 =1

2.2. Egyptian System

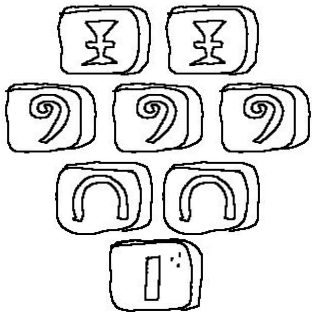
The Egyptian people used these symbols.

One. Hundred.

Ten. Thousand.

Examples:

2.321



2.3. Roman System

- ✓ Symbols: I=1, V=5, X=10, L=50, C=100, D=500, M=1.000
- ✓ Rules:
  - If the I comes after the V then you add it (VI is 6).
  - If the I comes before the V then you subtract it (IV is four).
  - You can subtract only one number from another.
  - You can subtract { I from V; I from X; X from L; X from C; C from D; C from M }
  - You can repeat a symbol three times, but not four.
  - A bar above a number indicates multiplication by 1000.

3. POSITIONAL NUMBER SYSTEM

Observe this number: 53.275.967
Fifty three million two hundred and seventy-five thousand nine hundred and sixty-seven.

TEN MILLIONS	MILLIONS	HUNDRED THOUSANDS	TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	UNITS
5	3	2	7	5	9	6	7

Observe the difference between thousand and thousand**s**, hundred and hundred**s** and ten and ten**s**:

- 2.327: Two thousand three hundred and twenty-seven.
- 2.327: Two thousand**s**, three hundred**s**, two ten**s** and seven units.

4. APPROXIMATING NUMBERS

Rounding makes numbers that are easier to work with. Rounded numbers are only approximate.
How to approximate numbers?

- ✓ If a number ends in 1, 2, 3 or 4 you erase that figure and write 0. (Round down) *Example*: 74 rounded to the nearest ten is 70.
- ✓ If the number ends in a digit of 5 or more erase that figure and write 0, then add one to the figure before. (Round up) *Example*: The number 88 rounded to the nearest ten would be 90.

5. OPERATIONS WITH NATURAL NUMBERS

5.1 Addition (terms addends, verb: add)

5 + 3 = 8 *five **plus** three **equals** eight*

Properties
- Commutative: 2 + 3 = 3 + 2
- Associative: (2 + 4) + 6 = 2 + (4 + 6)
- Additive identity element: 0

5.2 Subtraction (verb: *subtract sth from sth*)

*Fifty-four **minus** eighteen **equals** thirty-six*

$$\begin{array}{rl} 54 & \text{minuend} \\ -\ 18 & \text{subtrahend} \\ \hline 36 & \text{difference} \end{array}$$

Remember: subtrahend + difference = minuend

5.3 Multiplication (terms *factors*)

*2 · 5 = 10 two **times** five **equals** ten // two **multiplied by** five **equals** ten*

Properties:
- Commutative: $2 \cdot 3 = 3 \cdot 2$
- Associative: $(2 \cdot 4) \cdot 6 = 2 \cdot (4 \cdot 6)$
- Multiplicative identity element: 1
- Distributive property of multiplication over addition: $(5+6) \cdot 2 = 5 \cdot 2 + 6 \cdot 2$

5.4 Division

Dividend	*Divisor*
Remainder	*Quotient*

15 : 3 = 5 *fifteen **divided by** three **equals** 5*

If *Remainder* = 0, it is called "Exact division".
Remember: Dividend = Divisor · Quotient + Remainder

6. ORDER OF OPERATIONS

1. **BRACKETS**
2. **OTHERS**
3. **DIVISIONS**

BODMAS.

4. **MULTIPLICATIONS**
5. **ADDITIONS**
6. **SUBSTRACIONS**

WORKSHEET

1. Write with Roman numbers:

   a) 2.345 =
   b) 749 =
   c) 5.728 =
   d) 12.325 =
   e) 49 =
   f) 1.449=

2. Write with decimal numbers:

   a) $\overline{\text{M}}$MCDXII =
   b) $\overline{\text{VI}}$CCCXXIV =
   c) DCCXLI =
   d) XXIV =
   e) MMMXCV =
   f) CDXXXVIII =

3. How do you read these numbers?

   a) 23.456 =
   b) 176.396 =
   c) 1.340.891 =
   d) 56.872 =
   e) 903 =
   f) 9.347 =

4. Partition these numbers:

   a) 341.208 = 3 Hundred Thousands + 4 Ten Thousands + 1 Thousands + 2 Hundreds + 0 Tens + 8 Units = 300.000 + 40.000 + 1.000 + 200 + 8

   b) 2.304.179 =

   c) 725.286=

5. Round to the nearest hundred:

   a) 2.586
   b) 43.231
   c) 72.472
   d) 163.748

6. Calculate:

   a) $(65 + 38) \cdot 2 - 19 =$
   b) $32 : (16 - 4 \cdot 3) + 1 =$
   c) $[2 \cdot (12 - 5 + 6) -1] + 19 =$
   d) $134 - 56 \cdot 2 + 123 : 3=$
   e) $34 \cdot 5 - 28 : 2 + 76 =$
   f) $13.876 + 651.494 + 7.531 - 32.816 =$

LESSON 1

WORD PROBLEMS

1. Peter *buy*s a farm for $643.750 and after *selling* it he makes a *profi*t of $75.250. How much does he *sell* it for?

2. With the money that I have now and $247 more, I can pay a *debt* of $525 and I still have $37. How much money do I have?

3. How many years are 6.205 days?

4. An airport has a plane *landing* every 10 minutes. How many planes land in one day?

5. There are 4500 *inhabitants* in a village, and there is a tree for every 90 *inhabitants*. How many trees are there in the village? How many trees have to be planted to have a tree for every 12 people?

6. Ann has 1187 € in the bank, she *spends* 385 € on a coat and 163 € on a dress, how much money does she have in the bank now?

Vocabulary

Farm – granja	Sell – vender	Profit – beneficio
Debt – deuda	Land – aterrizar	Inhabitant – habitante
Buy – comprar	Spend – gastar	

LESSON 2: POWERS AND ROOTS

Keywords

Power Root Base Exponent
SQUARED Cubed RADICAND Perfect square

1. WHAT ARE POWERS?

$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ Two ***to the power*** of 5 or two ***to the fifth***

Base → 2^5 ← Exponent

Special powers:

- 5^2 five ***squared***
- 4^3 four ***cubed*** or four to the third

2. PROPERTIES OF POWERS

1. $a^0 = 1$
2. $a^1 = a$
3. The power of a product is the product of the powers.

$$(a \cdot b)^n = a^n \cdot b^n$$

Example: $(2 \cdot 3)^4 = 2^4 \cdot 3^4$

4. The power of a quotient is the quotient of the powers.

$$(a : b)^n = a^n : b^n$$

Example: $(2 : 3)^4 = 2^4 : 3^4$

5. When multiplying powers *of the same base*, you keep the same base and add the exponents.

$$a^n \cdot a^m = a^{n+m}$$

Example: $5^3 \cdot 5^4 = 5^7$

6. When dividing powers *of the same base*, you keep the same base and subtract the exponents.

$$a^n : a^m = a^{n-m}$$

Example: $7^8 : 7^5 = 7^3$

7. When powering a power, you keep the base and multiply the exponents.

$$(a^n)^m = a^{n \cdot m}$$

Example: $(4^3)^2 = 4^6$

3. POWERS OF TEN

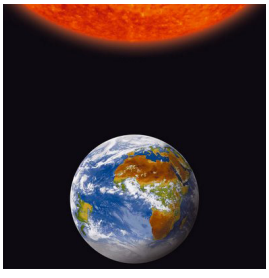
10 to the power of *n* is equal to 1 and *n* zeros
Examples: $10^5 = 100.000$
$10^7 = 10.000.000$

Using powers of ten you can:

- Partition a number as follows:

$$12.346 = 1 \cdot 10^4 + 2 \cdot 10^3 + 3 \cdot 10^2 + 4 \cdot 10 + 6$$

- And you can write long numbers in a short way.
  Exactly: $34.200.000.000 = 342 \cdot 10^8$
  Approximately: $687.430.863 \approx 687 \cdot 10^6$



4. SQUARE ROOTS

$\sqrt{a} = b \longrightarrow b^2 = a$
a ***radicand***
b ***root***
The opposite of squared a number is calculating its square root.
Example: $\sqrt{9} = 3$ The square root of nine equals 3

- A *perfect square* is a number that is the *square* of some other number.

  Example: 16 is a perfect square because $4^2 = 16$.

- If a number is not a perfect square you can approximate its square root.

  Example: 20 is not a perfect square but you know that
  $4^2 = 16$ and $5^2 = 25$
  as $16 < 20 < 25$, then $4 < \sqrt{20} < 5$. We say that 4 is the *whole square root* of 20.

WORKSHEET

1. Calculate:

   a) 2^5 =
   b) 3^4 =
   c) 5^3 =
   d) 10^5 =
   e) 4^3 =
   f) 2^{10} =

2. How do you read the powers in the previous exercise?

   a)
   b)
   c)
   d)
   e)
   f)

3. Complete:

   a) $2^{\square} = 64$
   b) $3^{\square} = 243$
   c) $5^{\square} = 625$
   d) $10^{\square} = 100.000$
   e) $9^{\square} = 81$
   f) $7^{\square} = 343$

4. Partition these numbers using powers of ten:

   a) 34.690.146 =

   b) 763.901 =

   c) 183.853.952 =

5. Calculate using properties of powers and express as just one power:

   a) $2^3 \cdot 2^5 \cdot 2$ =
   b) $(4^3)^2$ =
   c) $(7^2)^5 : 7^4$ =
   d) $3^5 \cdot 3^2 : 3$ =

6. Calculate mentally:

   a) $\sqrt{900}$ =
   b) $\sqrt{625}$ =
   c) $\sqrt{81}$ =
   d) $\sqrt{10000}$ =

7. Calculate the whole square root of these numbers:

   a) $\sqrt{71}$
   b) $\sqrt{105}$
   c) $\sqrt{20}$
   d) $\sqrt{53}$

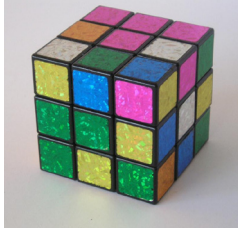
LESSON 2

WORD PROBLEMS

1. Calculate how many *tiles*, which *measure* one square metre, we need to *pave* the floor of a square patio. The *sides* of the patio are 22 metres long.

2. A square *garden* is 900 square metres. Calculate how long one *side* is.

3. How many mothers and fathers had your great-great-grandparents together?



4. Calculate how many unit cubes we can *fit* in a cube that has an *edge* of 10 units.

5. A square room has 2209 square *tiles*. How many *rows* are there?

Vocabulary

Tiles – baldosas	Measure – medir	Pave – pavimentar
Side – lado	Garden – jardín	Fit – encajar
Edge – arista	Row– fila	